Adaptively Modeling Cyber-Physical Systems with Applications in Change Detection

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September 2017

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Cyber-Physical Systems

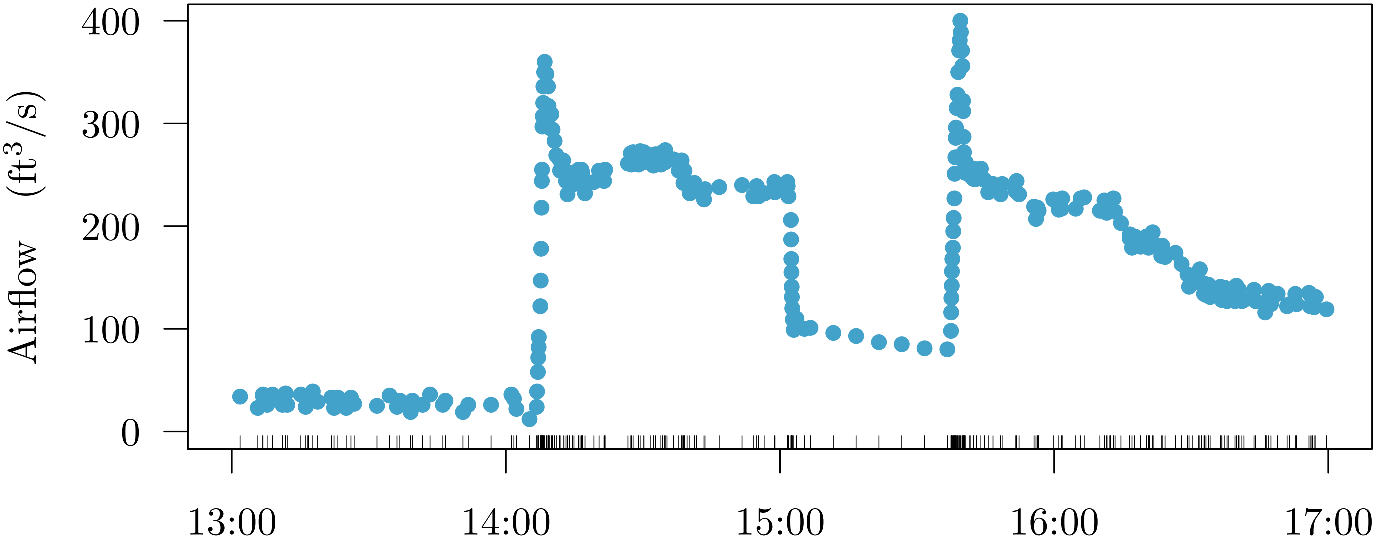
- A **cyber-physical system (CPS)** combines **cyber components** with **physical devices** to form complex systems.
- **Examples include:** smart cities, self-driving automobiles and office buildings.
- Issues with a CPS include **faults** and **intrusions** and can have severe consequences if they go undetected.
- This creates the need for a **flexible** approach for monitoring a CPS for faults and intrusions in **real-time**.

Talk Discusses:

- an **adaptive** modeling framework,
- a **change detection** method.

A Real World CPS

- The data analyzed comes from an **HVAC** system at **Los Alamos National Laboratories** (LANL).
- Eight story building equipped with **292 sensors** reporting communications between **building control units** and **variable air volumes**.



Adaptive Modeling Framework

Consider monitoring **bivariate** data streams of the form

$$\langle \boldsymbol{d}_1, \boldsymbol{d}_2, \ldots, \boldsymbol{d}_t, \ldots \rangle \qquad \boldsymbol{d}_t = (x_t, s_t),$$

physical reading

interarrival time

where $\boldsymbol{d}_t$ is modeled using a **time evolving normal gamma distribution** with density

$$f(\boldsymbol{d}_t|\theta_t) = \frac{\beta_t^{\alpha_t}\gamma_t^{1/2}}{\Gamma(\alpha_t)\sqrt{2\pi}}\, s_t^{\alpha_t - 1/2} \exp\left[-\beta_t s_t - \frac{\gamma_t s_t (x_t - \mu_t)^2}{2}\right],$$

and $\theta_t = (\mu_t, \gamma_t, \alpha_t, \beta_t)$.

Omitted: a **streaming validation procedure** which justifies this modeling assumption on the HVAC data.

Adaptive Modeling Framework

- The normal-gamma distribution has a lot of **desirable properties** that make it attractive for monitoring a CPS:
  - $X_t|S_t \sim \mathcal{N}(\mu_t, f(\gamma_t, s_t)) \implies$ model **dependence**,
  - $S_t \sim \Gamma(\alpha_t, \beta_t) \implies$ **flexibility** in monitoring interarrival times.

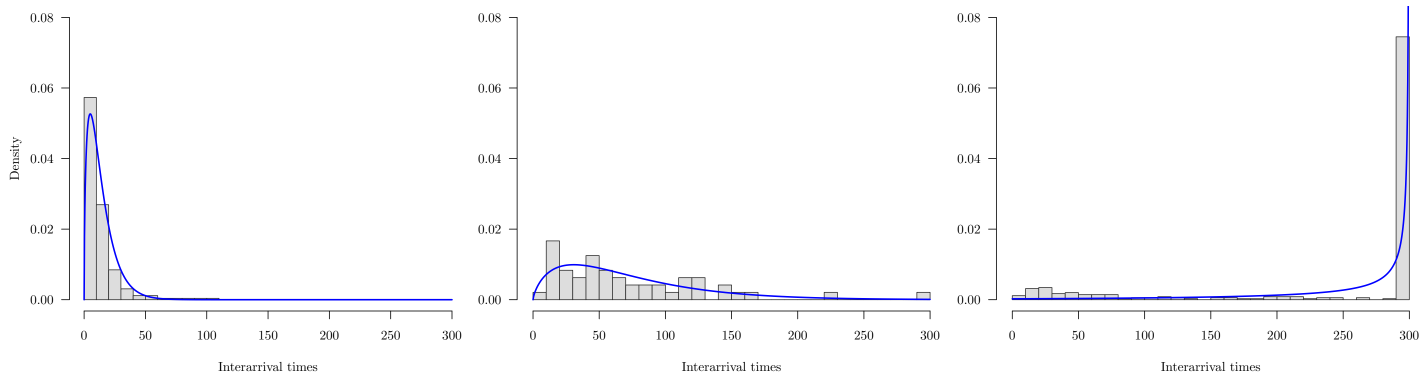


Figure 1: Histograms of the interarrival time distributions for three airflow sensors. **Blue lines** correspond to **gamma densities** fitted via maximum likelihood estimation to show **flexibility**.

Adaptive Parameter Estimation

Need a way to **efficiently** and **adaptively** estimate θ_t.

real-time

drift

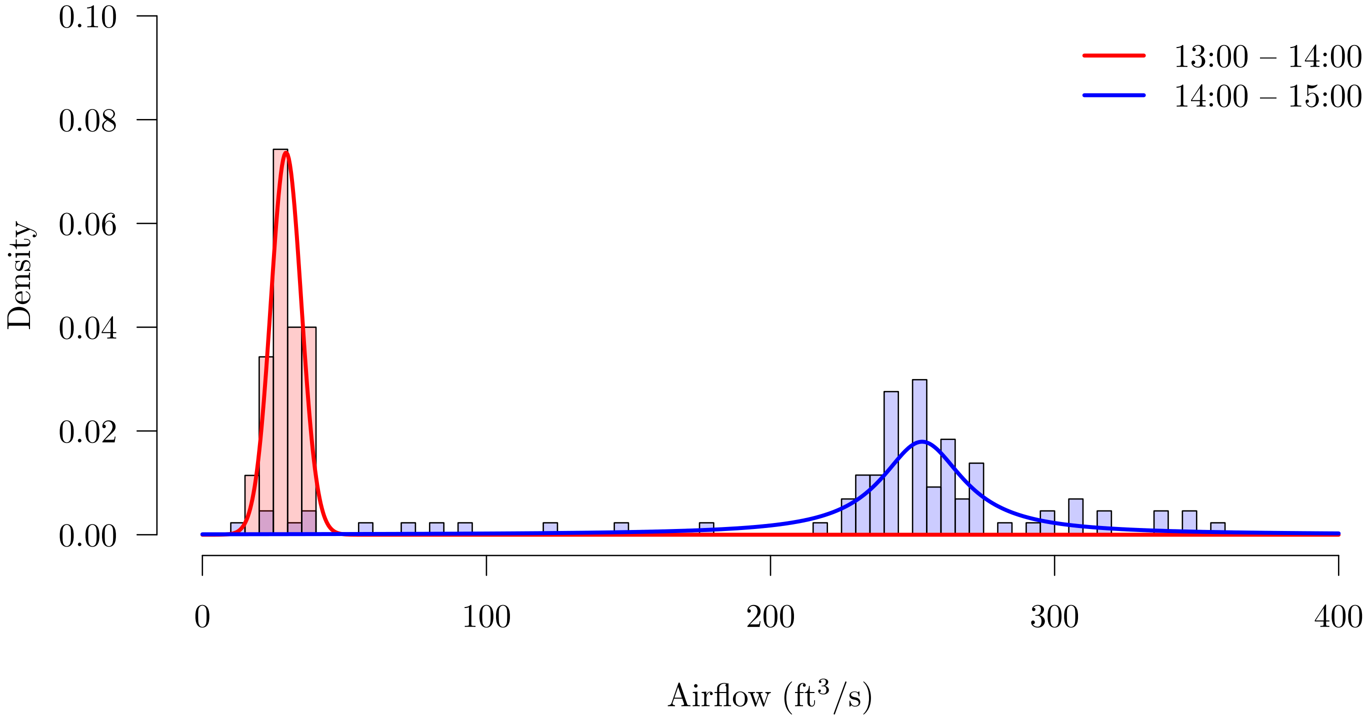


Figure 2: Illustration showing how airflow distributions **drift**.

Adaptive parameter estimation

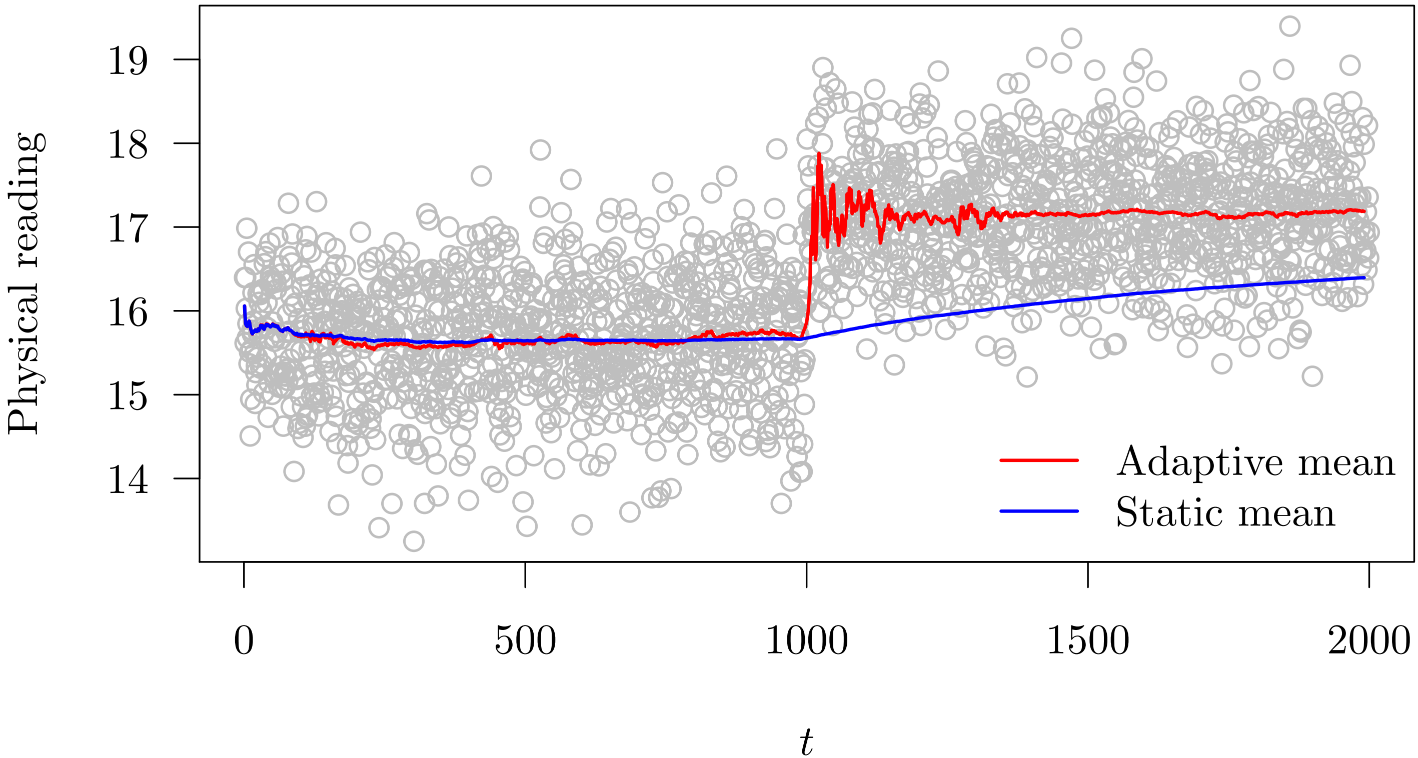
- **Idea:** exponentially **down-weight** historic data as new data is observed using **forgetting factors**.
- Consider a **weighted log-likelihood function** of the form:

$$\mathcal{L}_\lambda(\theta \mid \boldsymbol{d}_{1:t}) = \sum_{k=1}^{t} w_k \left[\alpha \log(\beta) + \frac{\log(\gamma)}{2} - \log\left(\Gamma(\alpha)\right) \right. \\ \left. + \left(\alpha - \frac{1}{2}\right) \log(s_k) - \beta s_k - \frac{\gamma s_k (x_k - \mu)^2}{2} \right],$$

- $w_k = \prod_{p=k}^{t-1} \lambda_p$ are the **weights** associated with the data,
- $\lambda_p \in [0, 1]$ is a **forgetting factor**.

Tuning the Forgetting Factors

The forgetting factors are tuned online via a **stochastic gradient descent step**.



Update Equations

Recursive updates:

$$\tilde{\mu}_t = \frac{m_{\boldsymbol{x}\boldsymbol{s},t}}{m_{\boldsymbol{s},t}}$$

$$\tilde{\gamma}_t = \frac{n_t}{\xi_t}$$

$$\tilde{\alpha}_t = \frac{(3-\zeta_t) + \sqrt{(\zeta_t - 3)^2 + 24\zeta_t}}{12\zeta_t}$$

$$\tilde{\beta}_t = \frac{\tilde{\alpha}_t n_t}{m_{\boldsymbol{s},t}}$$

Auxiliary updates:

$$n_t = \lambda_{t-1} n_{t-1} + 1$$

$$m_{f(\boldsymbol{x},\boldsymbol{s}),t} = \lambda_{t-1} m_{f(\boldsymbol{x},\boldsymbol{s}),t-1} + f(x_t, s_t)$$

$$\xi_t = m_{\boldsymbol{x}^2\boldsymbol{s},t} - \tilde{\mu}_t^2 m_{\boldsymbol{x}\boldsymbol{s},t}$$

$$\zeta_t = \log\left(\frac{m_{\boldsymbol{s},t}}{n_t}\right) - \frac{m_{\log(\boldsymbol{s}),t}}{n_t}$$

$$m_{f(\boldsymbol{x},\boldsymbol{s}),t} = \sum_{k=1}^{t} w_k f(x_k, s_k)$$

These updates do not require **any** observations prior to time t to maintain up-to-date estimates. This makes the adaptive modeling framework **efficient** for the data streaming from a CPS.

The Change Detector

Consider **two** normal-gamma distributions: one with **adaptive** parameter estimates $\tilde{\theta}_t$ and the other having **static** (usual MLEs) estimates $\hat{\theta}_t$. **The main idea:**

- during **stationary periods** $\tilde{\theta}_t$ and $\hat{\theta}_t$ should be **roughly the same**,
- when the stream experiences a **change** $\tilde{\theta}_t$ and $\hat{\theta}_t$ should **diverge**.

If this **dissimilarity** can be quantified, a change could be flagged whenever $\tilde{\theta}_t$ and $\hat{\theta}_t$ get "too far" apart.

The Change Detector

Use the **KL-Divergence** and flag a change whenever

$$\mathrm{KL}_t\left(f(\tilde{\theta}_t)||f(\hat{\theta}_t)\right) > \varepsilon_t.$$

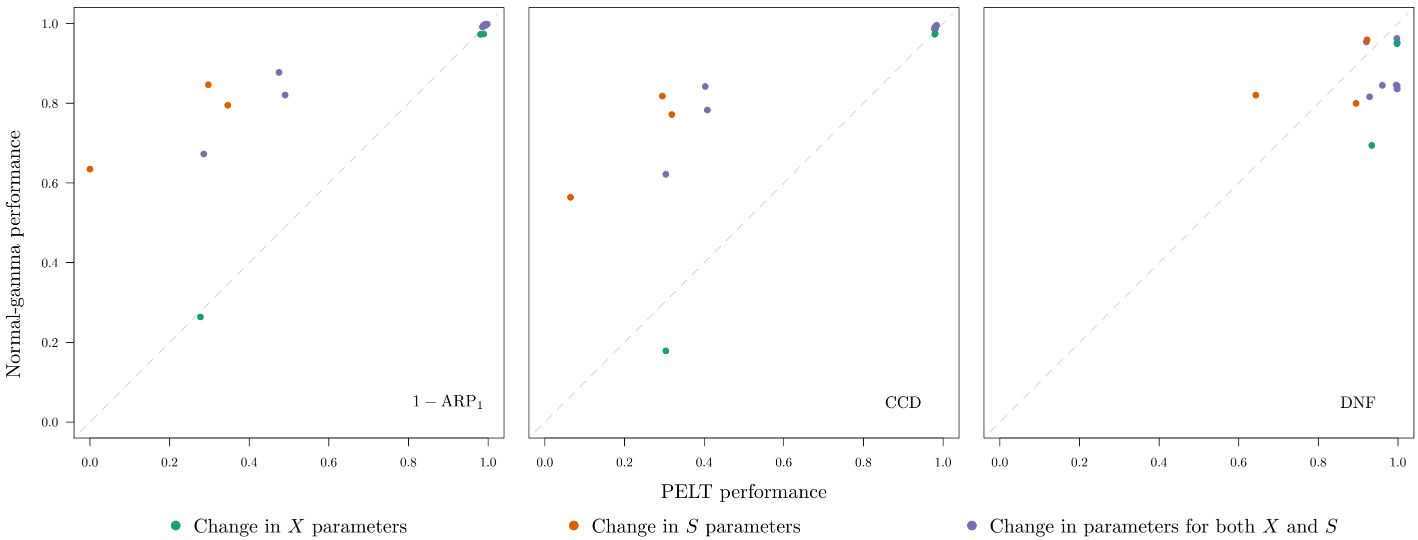
NG density

adaptive threshold

Simulation study:

- compare to R's implementation of **PELT** (Killick, Eckley 2014),
- consider changes in **all subsets** of $\theta_t = (\mu_t, \gamma_t, \alpha_t, \beta_t)$,
- use **average detection delay** and proportion of **change points correctly detected** and **detections that are not false** as **performance metrics**.

The Change Detector



- Given a performance measure, all points **above** the line $y = x$ indicates our method outperforming PELT.
- Our method, overall, outperforms PELT with respect to the **speed** (left panel) and **accuracy** of true dectections (center), but is marginally worse for false detections (right panel).

Conclusions

This talk has presented a **complete** framework for monitoring a **general** CPS that allows for:

- temporally **adaptive** and **efficient** parameter estimation,
- and **change detection**.

Accepted for publication:
"An Adaptive Modeling Framework for Bivariate Data Streams with Applications to Change Detection in Cyber-Physical Systems", ICDM DMCIS Workshop, 2017.

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Appendix:

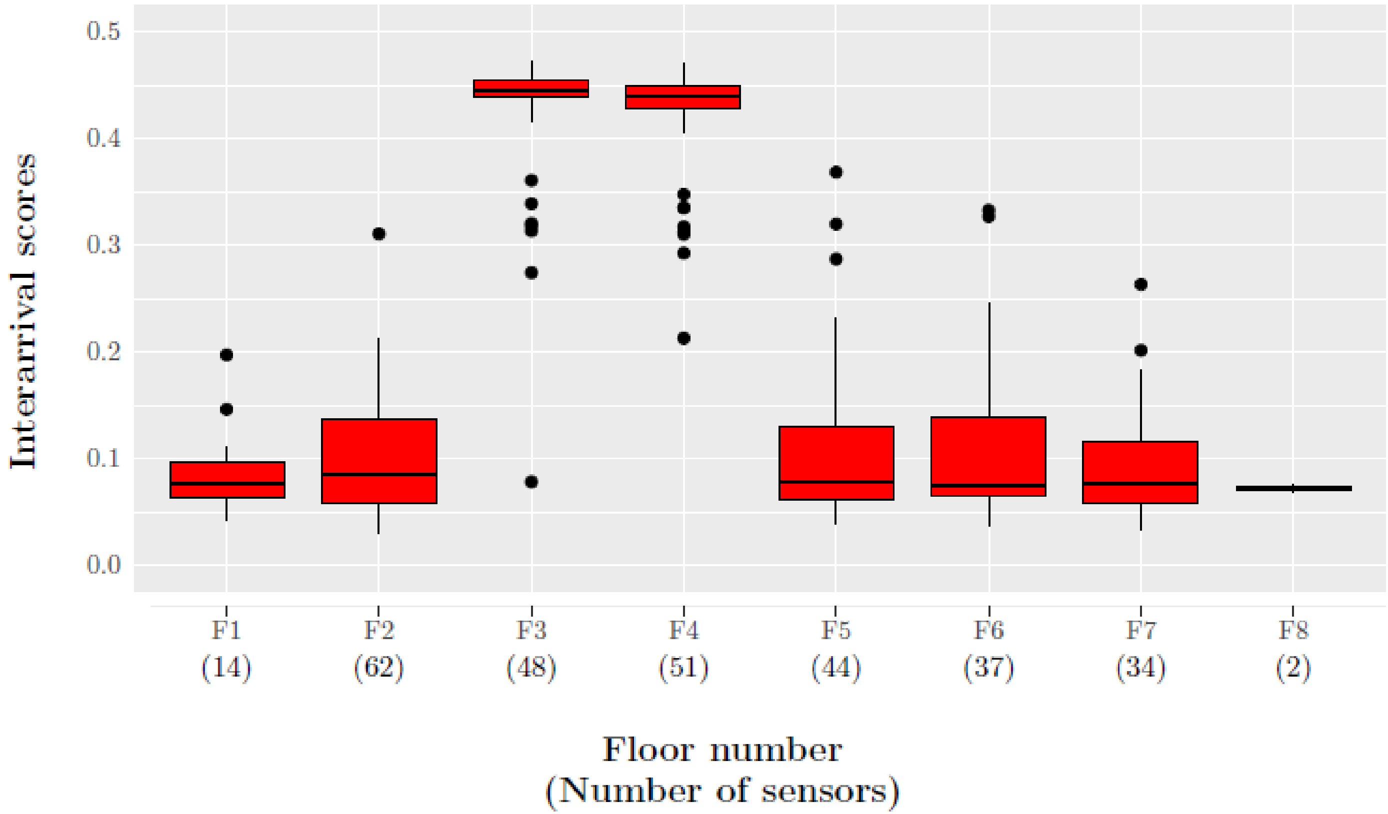
Model validation:

- Suppose we "think" a stream $v_{1:N}$ has CDF F_V parameterized by some vector θ.
- **Method idea:** apply the **PIT** sequentially to $v_{1:N}$ and compute a **KS score**.

The method

- For each $t = 1, \ldots, N$
  - **compute:** $\tilde{\theta}_t$,
  - **apply PIT:** $\tilde{u}_t = 1 - F_V(v_t \mid \tilde{\theta}_t)$.
- Compute **empirical CDF** $\tilde{F}_u$ using $\{\tilde{u}_t\}_{t=1}^N$.
- Return the **score** $\max_{u \in \{\tilde{u}_t\}_{t=1}^N} \left| \tilde{F}_U(u) - u \right|$.

Appendix



Interarrival scores
0.0
0.1
0.2
0.3
0.4
0.5
F1 (14)
F2 (62)
F3 (48)
F4 (51)
F5 (44)
F6 (37)
F7 (34)
F8 (2)
Floor number
(Number of sensors)